A Bipedal Walking Pattern Generator that Considers Multi-Body Dynamics by Angular Momentum Estimation

Johannes Mayr, Hubert Gattringer, and Hartmut Bremer

Johannes Kepler University Linz, Altenberger Strasse 69, 4040 Linz, Austria

{joh.mayr, hubert.gattringer, hartmut.bremer}@jku.at, +43 (732) 2468 - 6496

Objectives:

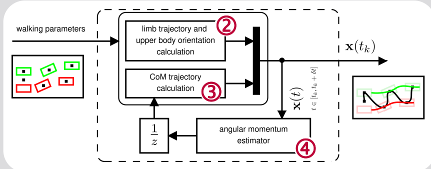
- stable walking patterns
- real time algorithm
- considering unilateral contacts
- considering multi-body dynamics

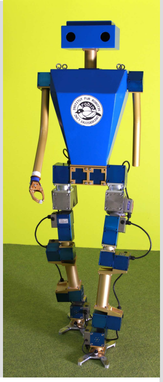
Challenges:

- high number of d.o.f. and complex structure
- sufficiently fast calculation time
- limited contact forces due to unilateral contact

Our Solution:

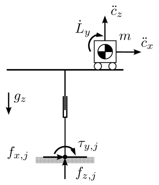
① **derivation of a model**
② **calculation of limb trajectories**
③ **calculation of center of mass trajectory**
④ **updating estimation of the rate of change of angular momentum**





① Dynamical Model

$$\begin{bmatrix} m\mathbf{I} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{M}_{\omega_B}(\mathbf{q}_J) & \mathbf{M}_{\omega_B,J}(\mathbf{q}_J) \\ \mathbf{0} & \mathbf{M}^T_{\omega_B,J}(\mathbf{q}_J) & \mathbf{M}_J(\mathbf{q}_J) \end{bmatrix} \begin{bmatrix} \ddot{\mathbf{r}}_C \\ \dot{\boldsymbol{\omega}}_B \\ \ddot{\mathbf{q}}_J \end{bmatrix} + \begin{bmatrix} -m\mathbf{g} \\ \mathbf{h}_{\omega_B}(\dot{\mathbf{s}},\mathbf{q}_J) \\ \mathbf{h}_J(\dot{\mathbf{s}},\mathbf{q}_J) \end{bmatrix} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \boldsymbol{\tau}_J \end{bmatrix} + \sum_j \lambda_j \begin{bmatrix} \mathbf{R}_j & \mathbf{0} \\ & \mathbf{J}^T_{\omega_B,j} \\ & \mathbf{J}^T_{J,j} \end{bmatrix} \begin{pmatrix} \mathbf{f}_j \\ \boldsymbol{\tau}_j \end{pmatrix}$$ [1]



$$m(\ddot{\mathbf{r}}_c - \mathbf{g}) = \sum_j \lambda_j \mathbf{R}_j \mathbf{f}_j$$

$$\dot{\mathbf{L}}_C = \sum_j \lambda_j \mathbf{R}_j \boldsymbol{\tau}_j + \sum_j \lambda_j (\mathbf{r}_j - \mathbf{r}_C) \times \mathbf{R}_j \mathbf{f}_j,$$

$$\dot{\mathbf{L}}_C = \mathbf{R}_B(\mathbf{M}_{\omega_B}\dot{\boldsymbol{\omega}}_B + \mathbf{M}_{\omega_B,J}\ddot{\mathbf{q}}_J + \mathbf{h}_{\omega_B})$$

rate of change of angular momentum

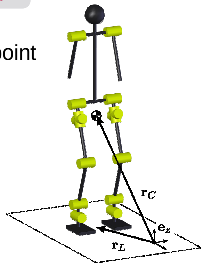
- total contact forces/torques at arbitrary contact point

$$\begin{bmatrix} \mathbf{f}_C \\ \boldsymbol{\tau}_C \end{bmatrix} = \begin{bmatrix} (\ddot{\mathbf{r}}_C - \mathbf{g})m \\ (\mathbf{r}_C - \mathbf{r}_L) \times (\ddot{\mathbf{r}}_C - \mathbf{g})m + \dot{\mathbf{L}}_C \end{bmatrix}$$

*

$\dot{\mathbf{L}}_C = \dot{\mathbf{L}}_C^{est}$
$\mathbf{g} = g\,\mathbf{e}_z$
$\mathbf{e}_z \cdot \mathbf{r}_j = const$

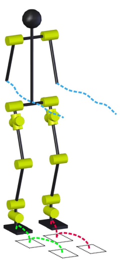
$$\begin{bmatrix} f_{c,x} \\ f_{c,y} \\ f_{c,z} \\ \tau_{c,x} \\ \tau_{c,y} \\ \tau_{c,z} \end{bmatrix} = \begin{bmatrix} m\ddot{c}_x \\ m\ddot{c}_y \\ m(\ddot{c}_z + g) \\ m(\ddot{c}_z+g)(c_y - l_y) - m\ddot{c}_y c_z + \dot{L}_x^{est} \\ -m(\ddot{c}_z+g)(c_x - l_x) + m\ddot{c}_x c_z + \dot{L}_y^{est} \\ m\ddot{c}_y(c_x - l_x) - m\ddot{c}_x(c_y - l_y) + \dot{L}_z^{est} \end{bmatrix}$$



② Limb Trajectories

- feet, arms and head trajectories
- upper-body orientation

generated according to the walking parameters

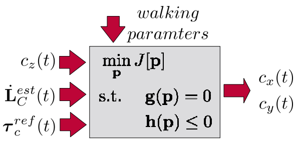


Experiment:

- upper-body is kept upright
- arms kept at constant position
- feet are kept parallel to the floor
- 5th order polynomials

③ Center of Mass Trajectory

- contact forces/torques depend on
  - center of mass trajectory
  - rate of change of angular momentum (estimated from previous cycle)
- parameterized as uniform B-splines
- parameters chosen to
  - minimize Euclidean norm of contact forces/torques
  - satisfy constraints



$$J[\mathbf{p}] = \frac{1}{2}\int_{t_k}^{t_k+\delta t} \sum_{i=\{x,y\}} \{\alpha_i \ddot{c}_i^2 + \beta_i(\tau_{c,i} - \tau_{c,i}^{ref})^2 + \gamma_i(c_i - p_i^{ref})^2\}dt$$

objective function

Experiment:

- cubic uniform B-splines (19 control points)
- sliding window size 1.6s
- planning interval 0.1s

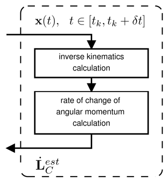
constraints:

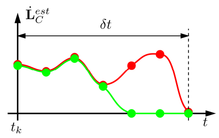
- CoM start constraints
- CoM terminal constraints
- contact force constraints (unilateral contact)

④ Update Angular Momentum Estimation

- estimation for next cycle by using:
  - walking pattern from current planning cycle
  - inverse kinematics (Newton method)
  - dynamical model from ①

  for n nodes in the sliding window
- interpolated by uniform B-splines
- choice of nodes:
  - i. n=0 → 3D-LIPM [2]
  - ii. uniformly distributed 🔴
  - iii. concentrated at the beginning 🟢





Experiment:

- linear uniform B-splines (17 control points)

Results and Conclusion

algorithm inputs:

- footstep locations
- walking parameters
- reference torques

algorithm outputs:

- foot trajectory
- center of mass trajectory
- upper body orientation
- ground reaction forces/torques

Experiments:

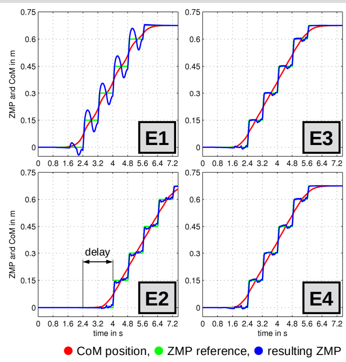
E1: 3D-LIPM (n=0) [2]
E2: 3D-LIPM with 2nd iteration [3]
E3: proposed scheme with uniformly distributed nodes (n=16)
E4: proposed scheme with nodes concentrated at the beginning (n=8)

Calculation times: (on an Intel i5-2500, 3.3 GHz, RTAI 3.81)

estimation (ES): 150µs per node**
quadratic program (QP): 80µs

Comparison:

	Delay in s	ZMP error in mm				ES calls	QP calls
		RMS x	RMS y	MAX x	MAX x		
E1	0	23.2	5.9	63.6	16	0	1
E2	1.6	4.9	4.5	16.3	14.6	1	2
E3	0	3.2	3.4	13.2	11	16	1
E4	0	3.3	3.8	11.9	12.8	8	1



[1] Wieber, Holonomy and nonholonomy in the dynamics of articulated motion, *Fast Motions in Biomechanics and Robotics, Springer,* 2005
[2] Kajita et al., The 3D linear inverted pendulum mode: A simple modeling for a biped walking pattern generation, IROS 2001
[3] Kajita et al., Biped walking pattern generation by using preview control of zero-moment point, ICRA 2003

Support of the present work in the framework of the peer-reviewed Austrian Center of Competence in Mechatronics (ACCM) is gratefully acknowledged.

* Assumptions are made for the sake of clarity and are not mandatory. ** Solves the complete inverse dynamics and can be improved significantly.

INSTITUTE FOR ROBOTICS · robin · LINZ UNIVERSITY

ACCM · AUSTRIAN CENTER OF COMPETENCE IN MECHATRONICS

JOHANNES KEPLER UNIVERSITY LINZ | JKU